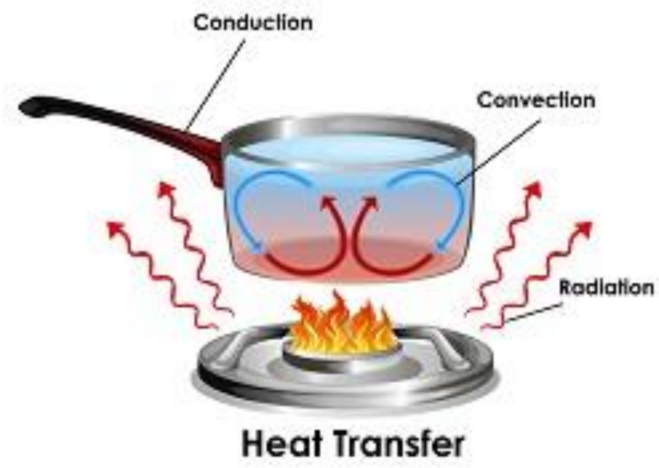
DEPARTMENT OF MECHANICAL ENGINEERING

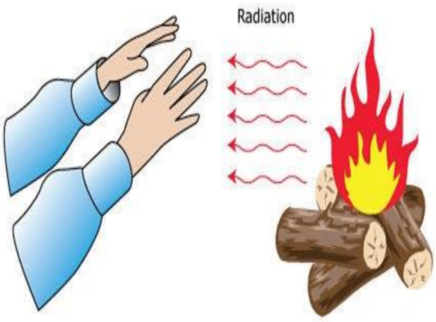
ABIT

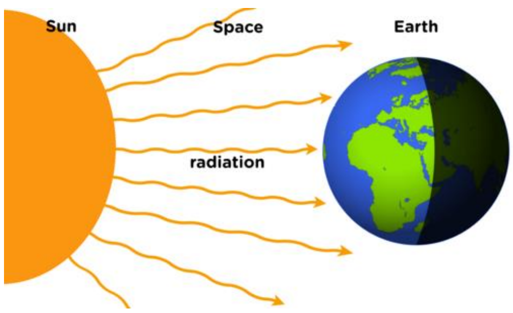
HEAT TRANSFER



MODULE-III

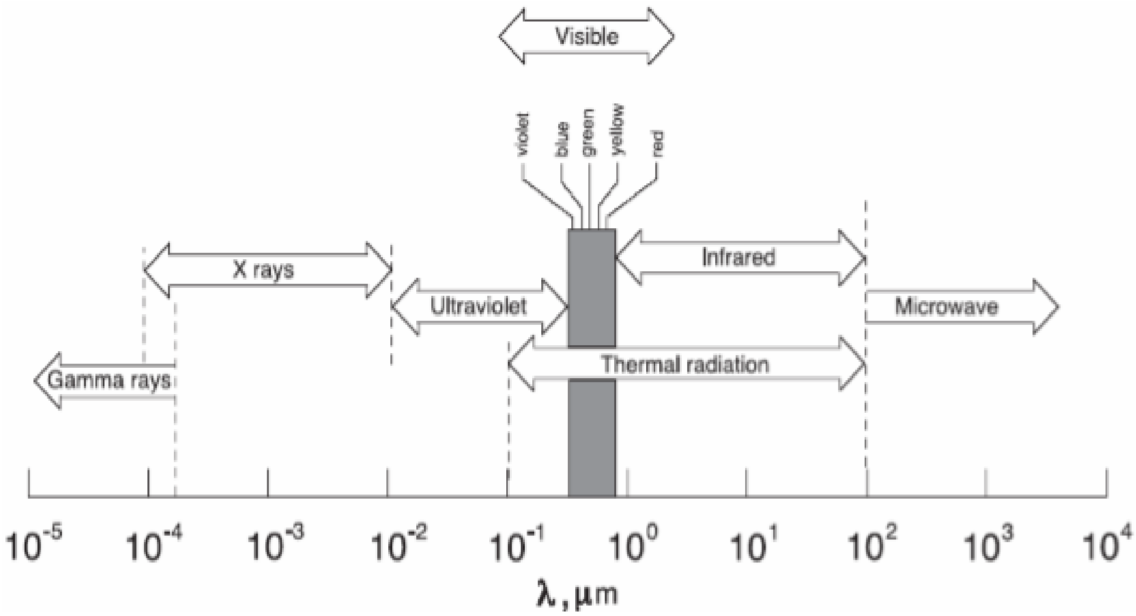
Radiative heat exchange

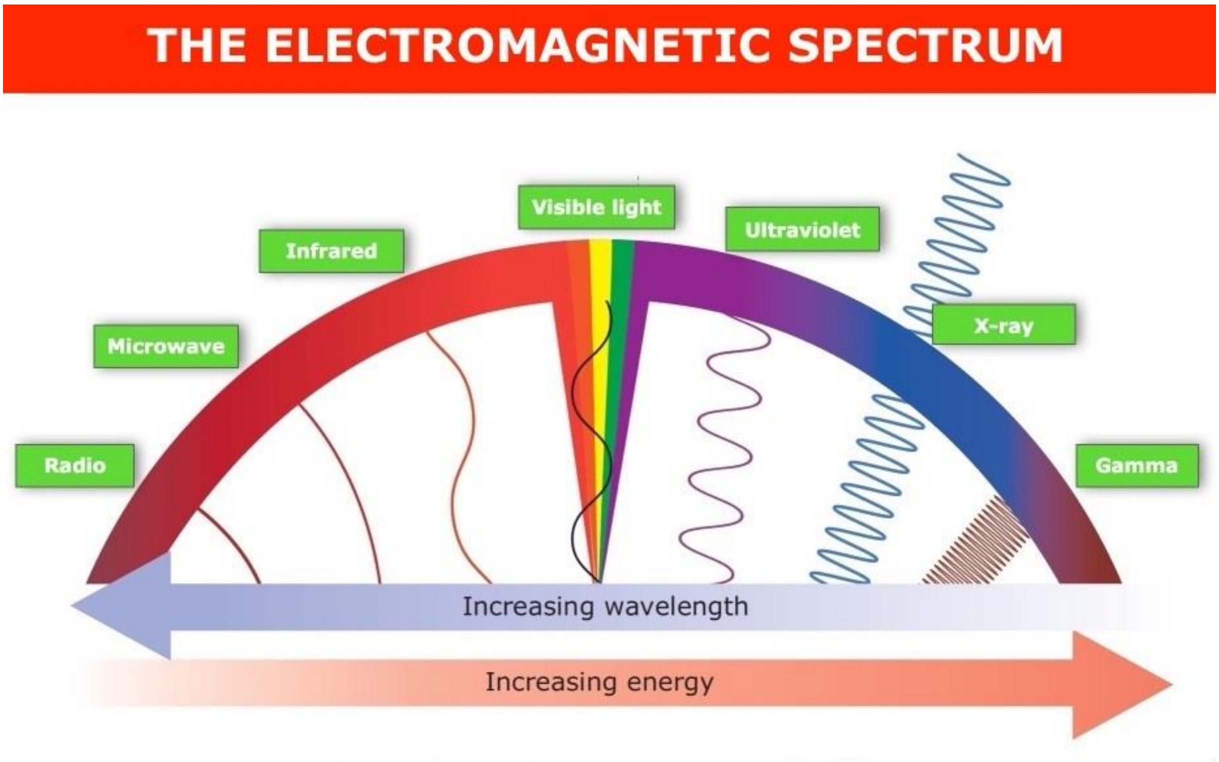




INTRODUCTION

- Radiation heat transfer is defined as “the transfer of energy across a system boundary by means of an electromagnetic mechanism which is caused mainly by a temperature difference.”
- Whereas the heat transfer by conduction and convection takes place only in the presence of medium, radiation heat transfer does not require a medium. It occurs most effectively in vacuum.
- The rate of heat transfer by conduction and convection varies as the temperature difference to the first power, whereas the radiant heat exchange between two bodies depends on the difference between their temperatures to the fourth power.
- Both the amount of radiation and the quality of radiation depend upon temperature.
- The energy which a radiating surface releases is not continuous but is in the form of successive and separate packet or quanta of energy called photons.
- The photons are propagated through space as rays , the movement of photons is described as electromagnetic waves.
- The photons travel (with speed equal to that of light) in straight path with unchanged frequency, when they approach the receiving surface , there occurs reconversion of wave motion into thermal energy which is partly absorbed , reflected or transmitted through the receiving.
- All types of electromagnetic waves are classified in terms of wavelength
- The electromagnetic spectrum is shown in the figure below.



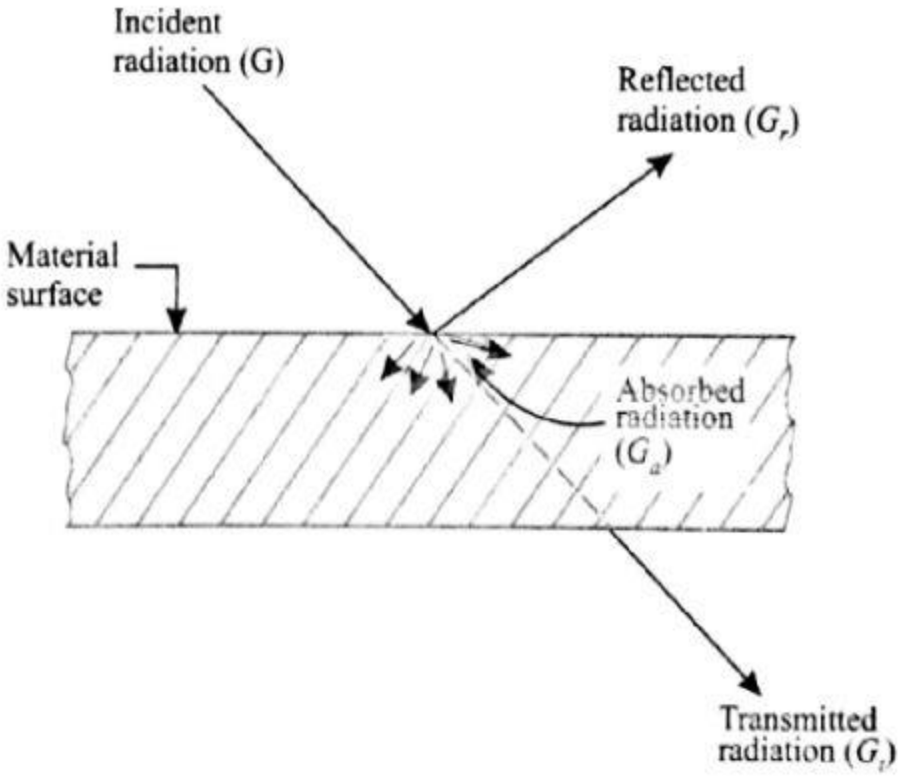


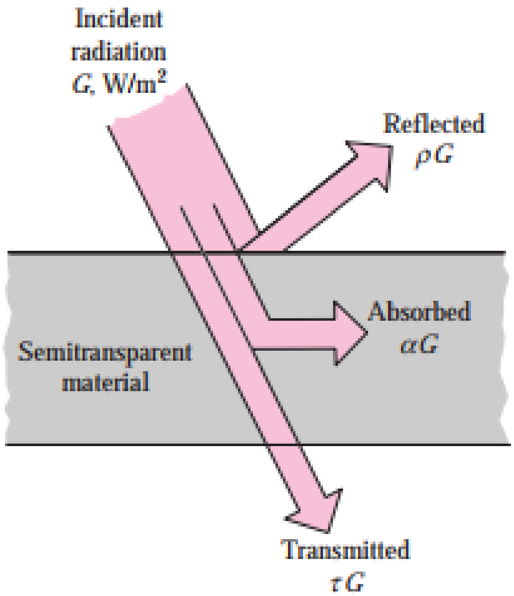
- The emission of thermal radiation (range lies between wavelength of 10^{-7}m and 10^{-4}m) depends upon the nature , temperature and state of emitting surface; however , with gases the dependence is also upon the thickness of the emitting layer and the gas pressure.
- Thermal radiations exhibit characteristics similar to those of visible light, and follow optical laws. These can be reflected, refracted and are subjected to scattering and absorption when they pass through a media.

Electromagnetic radiation-link below

https://www.youtube.com/watch?v=cfXzwh3KadE

ABSORPTIVITY, REFLECTIVITY AND TRANSMISSIVITY





- When incident radiation (G) also called irradiation (defined as the total incident radiation on a surface from all directions per unit time and per unit area of surface ; expressed in W/m^2 and denoted by (G) impinges on a surface, three things happen, a part is reflected back (G_r), a part is transmitted through (G_t) and the remainder is absorbed (G_a) depending upon the characteristics of the body as shown in the figure above.
- By the conservation of energy principle,

$$G_a + G_r + G_t = G$$

Dividing both sides by G, we get

$$\frac{G_a}{G} + \frac{G_r}{G} + \frac{G_t}{G} = \frac{G}{G}$$

$$\alpha + \rho + \tau = 1$$

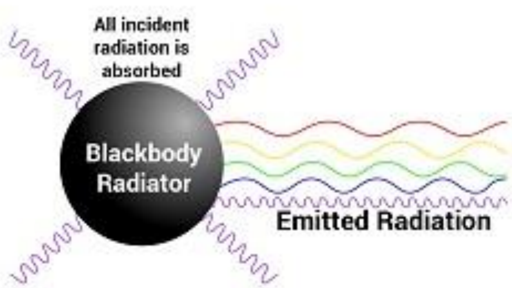
Where α= absorptivity (or fraction of incident radiation absorbed)

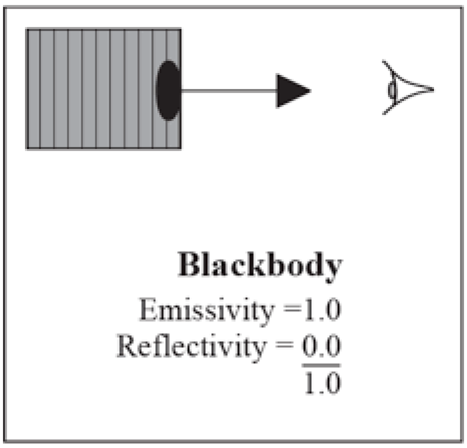
P= reflectivity (or fraction of incident radiation reflected), and

T= transmittivity (or fraction of incident radiation transmitted).

- When the incident radiation is absorbed , it is converted into internal enrgy.

Black body

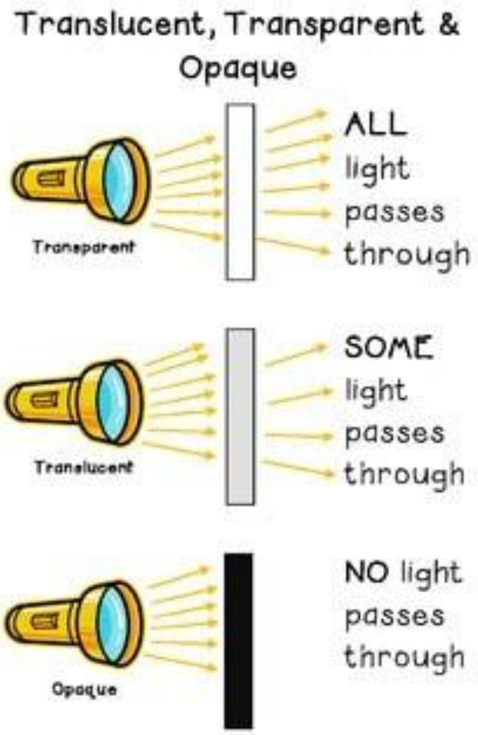




- For perfectly absorbing body , α=1, ρ0 , τ=0. Such a body is called a black body.
- In practice , a perfect black body (α=1) does not exist.

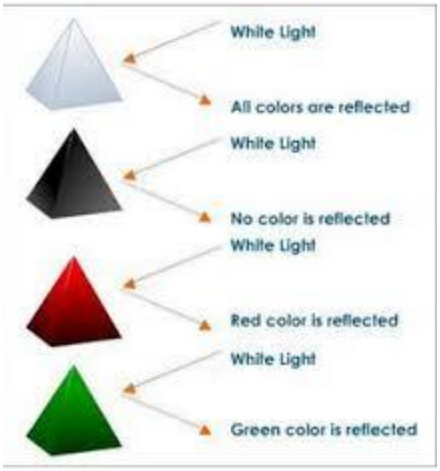
Opaque body

- When no incident radiation is transmitted through the body, it is called an opaque body.
- For the opaque body $\tau=0$ and $\alpha+\rho=1$
- Metals are considered to be opaque.

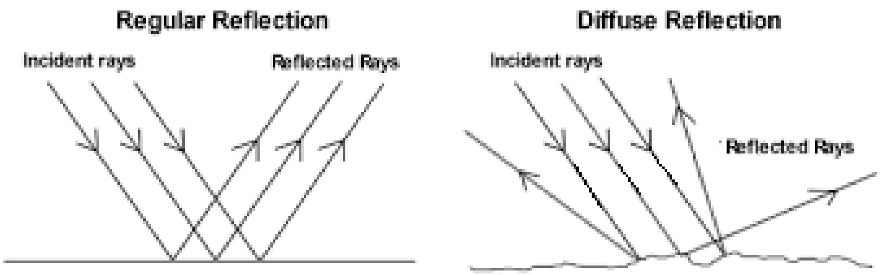


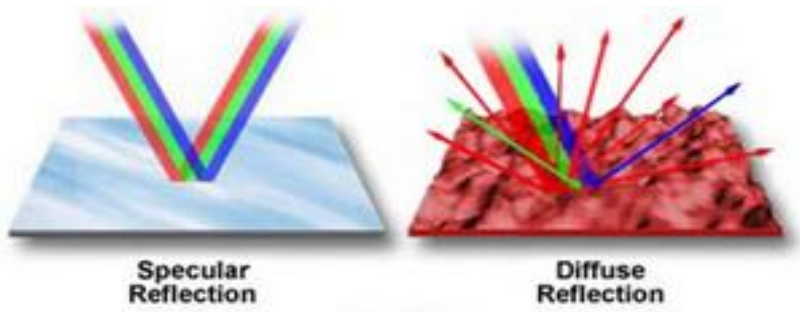
Whit body

- If all the incident radiation falling on the body are reflected, it is called a white body.
- For a white body , $\rho=1$, $\alpha=\tau=0$
- Gases such as hydrogen, oxygen and nitrogen (and their mixtures such as air) have a transmissivity of practically unity.



Reflections





1. Regular or specular reflection

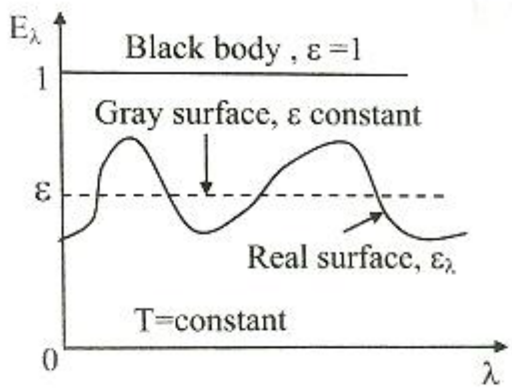
- Regular reflection implies that angle between the reflected beam and normal to the surface equals the angle made by the incident radiation with the same normal.
- Reflections from highly polished and smooth surfaces approaches specular characsteristics.

2. Diffused reflection

- In diffused reflection , the incident beam is reflected in all directions.
- Most of the engineering materials have rough surfaces and these rough surfaces give diffused reflections.

Gray body

- If the radiative properties , α, ρ and τ of a body are assumed to be uniform over the entire wavelength spectrum, then such a body is called gray body.
- It is also defined as one whose absorptivity of a surface does not vary with temperature and wavelength of radiation[$\alpha=\alpha_\lambda$= constant].
- A coloured body is one whose absorptivity of a surface varies with wavelength of radiation. $[\alpha \neq \alpha_\lambda]$



Emissivity(ϵ)

- It is defined as the ability of the surface of a body to radiate heat.
- It is also defined as the ratio of the emissive power of any body to the emissive power of a black body of equal temperature.

$$\varepsilon = \frac{E}{E_b}$$

- Its values vary between 0 to 1.

ABSORPTIVITY, REFLECTIVITY AND TRANSMISSIVITY –LINK BELOW

https://www.youtube.com/watch?v=dCqSnxRmxaw&list=PL03n4PEXL4saSbwpg0ygt-ef0NxYfaejI&index=17&t=0s

Emissivity link below

https://www.youtube.com/watch?v=lAxfvAwNYIo

KIRCHHOFF'S LAW

GUSTAV KIRCHHOFF-LINK- https://en.wikipedia.org/wiki/Gustav_Kirchhoff

- The law states that at any temperature the ratio of total emissive power E to the total absorptivity α is a constant for all substances which are in thermal equilibrium with their environment.

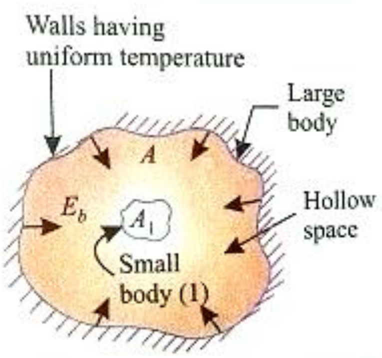


Fig. Derivation of Kirchhoff's law.

- Let us consider a large radiating body of surface area A which encloses a small body 1 of surface area A_1 .
- Let the energy fall on the unit surface of the body at the rate E_b. Of this energy , generally a fraction α will be absorbed by the small body.
- Thus , this energy absorbed by the small body 1 is $\alpha_1 A_1 E_b$, in which α_1 is the absorptivity of the body.
- When thermal equilibrium is attained , the energy absorbrd by the body 1 must be equal to the energy emitted, say E_1 per unit surface.
- Thus at equilibrium

$$A_1 E_1 = \alpha_1 A_1 E_b \dots\dots\dots\dots\dots\dots\dots(1)$$

- Now we remove body 1 and replace it by body 2having absorptivity α_2. The radiative energy impinging on the surface of this body is again E_b. We may write

$$A_2 E_2 = \alpha_2 A_2 E_b \dots\dots\dots\dots\dots\dots\dots(2)$$

- By considering generality of bodies, we obtain

$$E_b = \frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E}{\alpha} \dots\dots\dots\dots\dots\dots\dots\dots(3)$$

- Also as per definition of emissivity ϵ, we have

$$\varepsilon = \frac{E}{E_b}$$

Or,

$$E_b = \frac{E}{\varepsilon} \dots\dots\dots\dots\dots\dots\dots\dots\dots(4)$$

- By comparing equations (3) and (4), we obtain

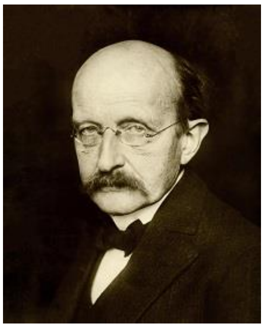
$$\epsilon = \alpha$$

- Thus , Kirchhoff's law also states that the emissivity of a body is equal to its absorptivity when the body remains in thermal equilibrium with its surroundings.

KIRCHHOFF'S LAW-LINK BELOW

https://www.youtube.com/watch?v=tFR5KUBg25E

PLANCK'S LAW



Max Planck-https://en.wikipedia.org/wiki/Max_Planck

- According to planck the spectral distribution of the radiation intensity of a black body is given by

$$(E_\lambda)_b = \frac{2\pi c^2 h \lambda^{-5}}{\exp\left(\frac{ch}{\lambda kT}\right) - 1} \text{......................(1)}$$

Where, $(E_\lambda)_b$ = monochromatic (single wave length) emissive power of a black body,

C=velocity of light in vacuum = 3 x 10^8 m/s,

h= Planck's constant=6.625 x 10^{-34}J.s,

λ = wavelength, µm,

k= Boltzmann constant =1.3805 x 10^{-23} J/K, and

T = absolute temperature, K

Hence the unit of $(E_\lambda)_b$ is W/m^2. µm.

- Planck's law is also written as

$$(E_\lambda)_b = \frac{C_1 \lambda^{-5}}{\exp\left(\frac{C_2}{\lambda T}\right) - 1} \text{..................(2)}$$

Where,

$$C_1 = 2\pi c^2 h = 3.742x10^8 W.\mu m^4 / m^2$$

$$C_2 = \frac{ch}{k} = 1.4388x10^4 \mu mK$$

- The quantity monochromatic emissive power , is defined as the energy emitted by the black surface in all directions at a given wavelength λ per unit wavelength interval around λ.
- The total and monochromatic emissive power are related by the equation,

$$E_b = \int_0^{\infty} (E_\lambda)_b \, d\lambda$$

- A plot of $(E_\lambda)_b$ is a function of temperature and wavelength is given in figure below.

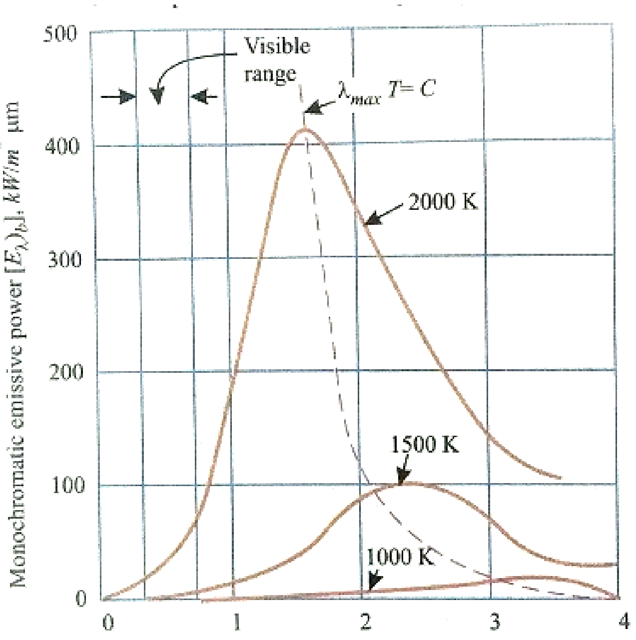
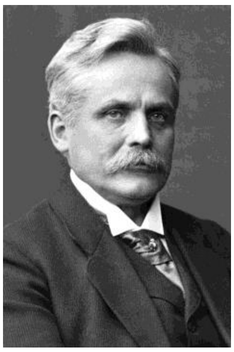


Fig. Variation of emissive power with wavelength.

WIEN'S DISPLACEMENT LAW



Wilhelm Wien- LINK-https://en.wikipedia.org/wiki/Wilhelm_Wien

- Wien established a relationship between the temperature of a black body and the wavelength at which the maximum value of monochromatic emissive power occurs.
- A peak monochromatic emissive power occurs at a particular wavelength.
- Wien's displacement law states that the product of λ_{max} and T is constant. i.e.,

$$\lambda_{\max} T = constant$$

$$(E_\lambda)_b = \frac{C_1\lambda^{-5}}{\exp\left(\frac{C_2}{\lambda T}\right)-1}$$

$(E_\lambda)_b$ becomes maximum (if T remains constant) when

$$\frac{d(E_\lambda)_b}{d\lambda} = 0$$

Or,

$$\frac{d(E_\pi)_b}{d\lambda} = \frac{d}{d\lambda}\left[\frac{C_1\lambda^{-5}}{\exp\left(\frac{C_2}{\lambda T}\right)-1}\right] = 0$$

Or,

$$\frac{\left[\exp\left(\frac{C_2}{\lambda T}\right)-1\right](-5C_1\lambda^{-6}) - C_1\lambda^{-5}\left\{\exp\left(\frac{C_2}{\lambda T}\right)\frac{C_2}{T}\left(-\frac{1}{\lambda^2}\right)\right\}}{\left[\exp\left(\frac{C_2}{\lambda T}\right)-1\right]^2} = 0$$

Or,

$$-5C_1\lambda^{-6}\exp\left(\frac{C_2}{\lambda T}\right) + 5C_1\lambda^{-6} + C_1C_2\lambda^{-5}\frac{1}{\lambda^2 T}\exp\left(\frac{C_2}{\lambda T}\right) = 0$$

Dividing both sides by $5C_1\lambda^{-5}$, we get

$$-\exp\left(\frac{C_2}{\lambda T}\right) + 1 + \frac{1}{5}C_2\frac{1}{\lambda T}\exp\left(\frac{C_2}{\lambda T}\right) = 0$$

Solving this equation by trial and error method, we get

$$\frac{C_2}{\lambda T} = \frac{C_2}{\lambda_{max}T} = 4.965$$

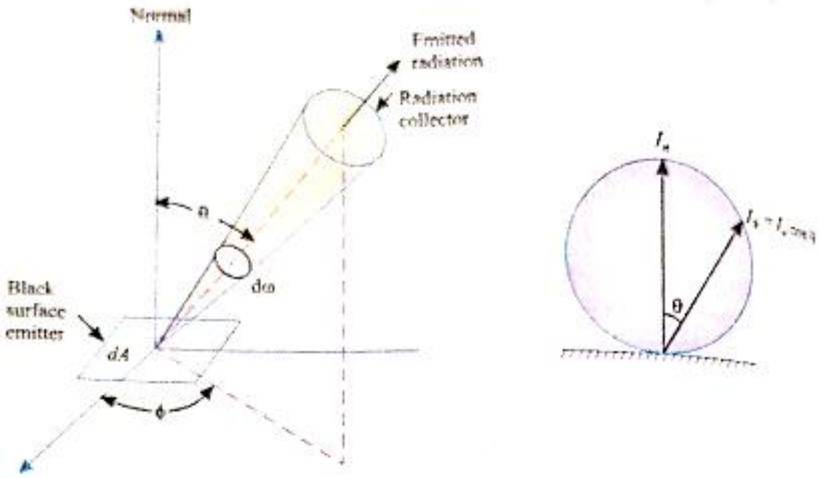
Hence,

$$\lambda_{max}T = \frac{C_2}{4.965} = \frac{1.439X10^4}{4.965}\mu mK = 2898\mu mK$$

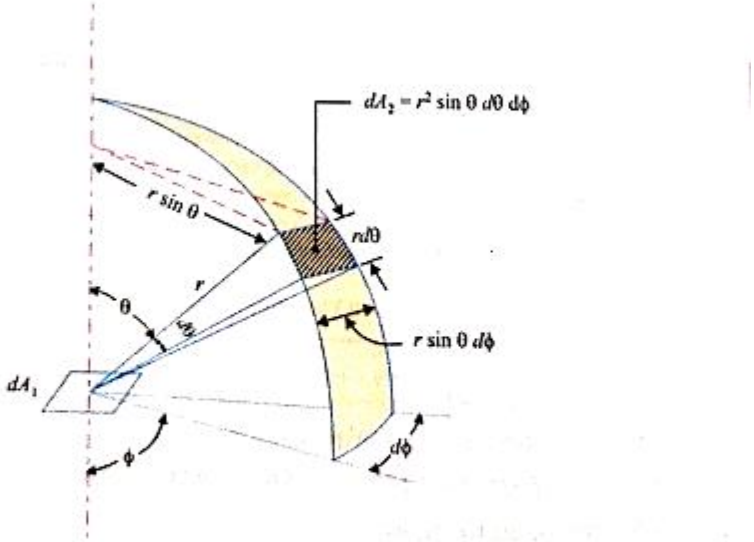
INTENSITY OF RADIATION

- When a surface element emits radiation, all of it will be intercepted by a hemispherical surface over the element. The intensity of radiation(I) is defined as the rate of energy leaving a surface in a given direction per unit solid angle per unit area of the emitting surface normal to the mean direction in space .
- A solid angle is defined as a portion of the space inside a sphere enclosed by a conical surface with the vertex of the cone at the centre of sphere..
- It is measured by the ratio of the spherical surface enclosed by the cone to the square of the radius of the sphere, its unit is steradian (sr).
- The solid angle subtended by the complete hemisphere is given by : $\frac{2\pi r^2}{r^2} = 2\pi$.
- Figure(a) shows a small black surface of area dA emitting radiation in different directions. A black body radiation collector through which the radiation pass is located at an angular position characterized by zenith angle θ towards he surface normal and angle ϕ of a spherical coordinate system. Further the collector subtends a solid angle dω when viewed from a point on the emitter.
- Let us now consider radiation from the elementary area dA_1 at the centre of a sphere as shown in figure (b).
- Suppose this radiation is absorbed by a second elemental area dA_2 , a portion of the hemispherical surface.
- The projected area of dA_1 on a plane perpendicular to the line joining dA_1 and dA_2 = $dA_1 \cos\theta$.
- The solid angle subtended by $dA_2 = \frac{dA_2}{r^2}$
- Hence the intensity of radiation $I = \frac{dQ_{1-2}}{dA_1 \cos\theta X \frac{dA_2}{r^2}}$

Where , dQ_{1-2} is the rate of radiation heat transfer from dA_1 to dA_2.



(a) Spacial distribution of radiations emitted from a surface



(b) Illustration for evaluating area dA_2

Fig. Radiation from an elementary surface.

- From figure(b), $$dA_2 = rd\theta(r\sin\theta d\phi)$$

  Or, $$dA_2 = r^2 \sin\theta . d\theta . d\phi$$

- Hence, $$dQ_{1-2} = IdA_1 . \sin\theta . \cos\theta . d\theta . d\phi$$

- The total radiation through the hemisphere is given by,

$$Q = IdA_1 \int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{\phi=0}^{\phi=2\pi} \sin\theta.\cos\theta.d\theta.d\phi$$

$$= 2\pi IdA_1 \int_{\theta=0}^{\theta=\frac{\pi}{2}} \sin\theta.\cos\theta.d\theta$$

$$= \pi IdA_1 \int_{\theta=0}^{\theta=\frac{\pi}{2}} \sin 2\theta.d\theta$$

Or, $Q = \pi IdA_1$

Also, $Q = EdA_1$

Hence, $EdA_1 = \pi IdA_1$

Or, $E = \pi I$

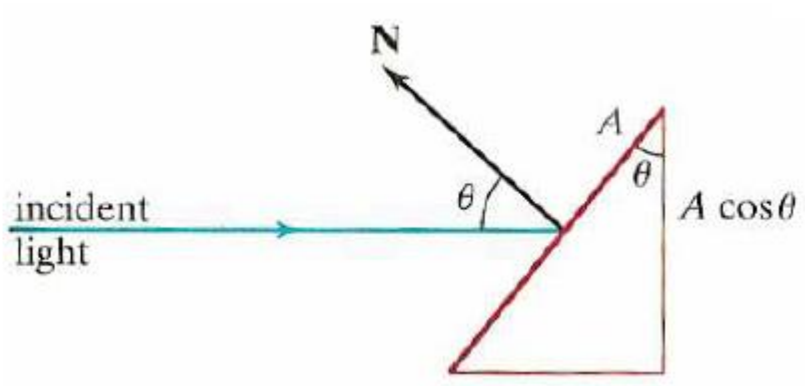
i.e. The total emissive power of a diffuse surface is equal to π times its intensity of radiation.

LAMBERT'SCOSINE LAW

Johann Heinrich Lambert-LINK-https://en.wikipedia.org/wiki/Johann_Heinrich_Lambert

- The law states that the total emissive power E_θ from a radiating plane surface in any direction is directly proportional to the cosine of the angle of emission.
- The angle of emission θ is the angle subtended by the normal to the radiating surface and the direction vector of emission of the receiving surface.
- If E_n be the total emissive power of the radiating surface in the direction of its normal , then

$$E_\theta = E_n \cos\theta$$



INTENSITY OF RADIATION AND LAMBERT'SCOSINE LAW-LINK BELOW

https://www.youtube.com/watch?v=E3558COemCU

RADIATION EXCHANGE BETWEEN BLACK BODIES SEPARATED BY A NON ABSORBING MEDIUM

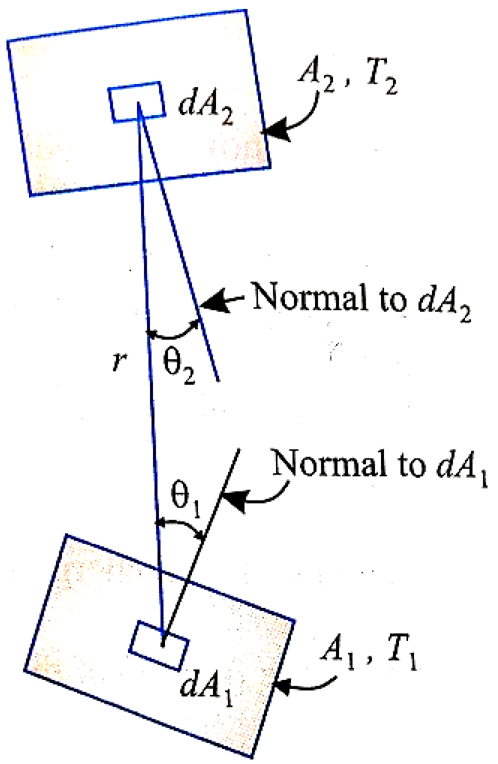


Fig. Radiation heat exchange between two black surface.

- Let us consider heat exchange between elementary ares dA_1 and dA_2 of two black radiating bodies, separated by a non absorbing medium, and having areas A_1 and A_2 and temperatures T_1 and T_2 respectively.
- The elementary areas are at a distance r apart and the normals to these areas make angles θ_1 and θ_2 with the line joining them.
- Each elemental area subtends a solid angle at the centre of other.
- Let $d\omega_1$ be subtended at dA_1 by dA_2 and $d\omega_2$ subtended at dA_2 by dA_1. Then

$$d\omega_1 = \frac{dA_2 \cos\theta_2}{r^2}$$

And $$d\omega_2 = \frac{dA_1 \cos\theta_1}{r^2} \text{ ....(1)}$$

- The energy leaving dA_1 in the direction given by the angle per unit solid angle

$$= I_{b_1} dA_1 \cos\theta_1$$

Where I_b =black body intensity, and

$dA_1 \cos\theta_1$ =projection of dA_1 on the line between the centres

- The rate of radiant energy leaving dA_1 and striking on dA_2 is given by.

$$dQ_{1-2} = I_{b_1} dA_1 \cos\theta_1 d\omega_1$$

$$= \frac{I_{b_1} \cos\theta_1 \cos\theta_2 dA_1 dA_2}{r^2} \text{......................(2)}$$

- This energy is absorbed by the elementary area dA_1 , since both the surfaces are black.
- The quantity of energy radiated by dA_2 and absorbed by dA_1 is given by

$$dQ_{1-2} = \frac{I_{b_2} \cos\theta_2 \cos\theta_1 dA_2 dA_1}{r^2} \text{................(3)}$$

- The net rate of transfer of energy between dA_1 and dA_2 is

$$dQ_{12} = dQ_{1-2} - dQ_{2-1}$$

$$= \frac{dA_1 dA_2 \cos\theta_1 \cos\theta_2}{r^2}(I_{b_1} - I_{b_2})$$

But, $$I_{b_1} = \frac{E_{b_1}}{\pi} \text{ and } I_{b_2} = \frac{E_{b_2}}{\pi} \text{ ....(4)}$$

Hence, $$dQ_{12} = \frac{dA_1 dA_2 \cos\theta_1 \cos\theta_2}{\pi r^2}(E_{b_1} - E_{b_2}) \ldots\ldots\ldots\ldots\ldots(5)$$

Or, $$dQ_{12} = \frac{\sigma dA_1 dA_2 \cos\theta_1 \cos\theta_2}{\pi r^2}(T_1^4 - T_2^4) \ldots\ldots\ldots\ldots\ldots(6)$$

- The rate of total net heat transfer for the total area A_1 and A_2 is given by

$$Q_{12} = \int dQ_{12} = \sigma(T_1^4 - T_2^4)\int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2 dA_1 dA_2}{r^2} \ldots\ldots\ldots(7)$$

- The rate of radiant energy emitted by A_1 that falls on A_2 is given by

$$Q_{1-2} = I_{b_1}\int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2 dA_1 dA_2}{r^2}$$

$$Q_{1-2} = \sigma T_1^4 \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2 dA_1 dA_2}{\pi r^2} \ldots\ldots\ldots\ldots(8)$$

- The rate of total energy radiated by A_1 is given by

$$Q_1 = A_1 \sigma T_1^4$$

- Hence the fraction of the rate of energy leaving area A_1 and impinging on area A_2 is given by

$$\frac{Q_{1-2}}{Q_1} = \frac{1}{A_1}\int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2 dA_1 dA_2}{\pi r^2} \ldots\ldots\ldots\ldots(9)$$

Or, $$\frac{Q_{1-2}}{Q_1} = F_{1-2} \ldots\ldots\ldots\ldots\ldots(10)$$

- F_{1-2} is known as 'configuration factor' or 'surface factor 'or 'view factor' between the two radiating surface and is a function of geometry only.
- Thus the shape factor may be defined as "The fraction of radiative energy that is diffused from one surface element and strikes the other surface directly with no intervening reflections:.
- Further, $$Q_{1-2} = F_{1-2} A_1 \sigma T_1^4 \ldots\ldots\ldots\ldots\ldots(11)$$
- Similarly, the rate of radiant energy by A_2 that falls on A_1 is given by

$$Q_{2-1} = \sigma T_2^4 \int_{A_1}\int_{A_2} \frac{\cos\theta_1 \cos\theta_2 dA_1 dA_2}{\pi r^2} \ldots\ldots\ldots\ldots(12)$$

- The rate of total energy radiated by A_2 is given by

$$Q_2 = A_2 \sigma T_2^4$$

- Hence the fraction of the rate of energy leaving area A_2 and impinging on area A_1 is given by

$$\frac{Q_{2-1}}{Q_2} = \frac{1}{A_2}\int_{A_1}\int_{A_2}\frac{\cos\theta_1\cos\theta_2 dA_1 dA_2}{\pi r^2}....................(13)$$

or,
$$\frac{Q_{2-1}}{Q_2} = F_{2-1}$$

- F_{2-1} is the shape factor of A_2 with respect to A_1.

$$Q_{2-1} = F_{2-1}A_2\sigma T_2^4......................(14)$$

- Hence we get, $A_1F_{1-2} = A_2F_{2-1}$......................(15)
- The above result is known as reciprocity theorem.
- Thus the net rate of heat transfer between two surfaces $A_1$and A_2 is given by

$$Q_{1-2} = A_1F_{1-2}\sigma(T_1^4 - T_2^4)$$

$$= A_2F_{2-1}\sigma(T_1^4 - T_2^4)......................(16)$$

*It may be noted that above equation is applicable to black surfaces only and must not be used for surfaces having emissivities different from unity.

RECIPROCITY THEOREM-LINK BELOW

https://www.youtube.com/watch?v=trSXhYeYV3c

- Geometrical factors for parallel planes (disc and rectangles) directly opposed and those for radiation between perpendicular rectangles with a common edge are shown in figures below.

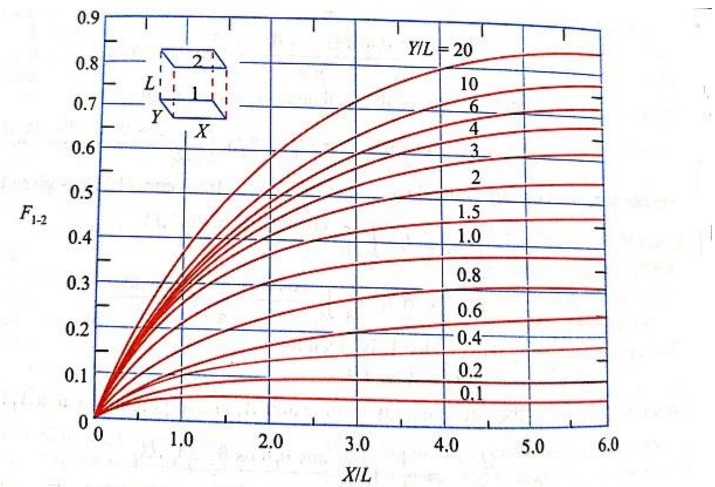


Fig. Shape factor for aligned parallel plates.

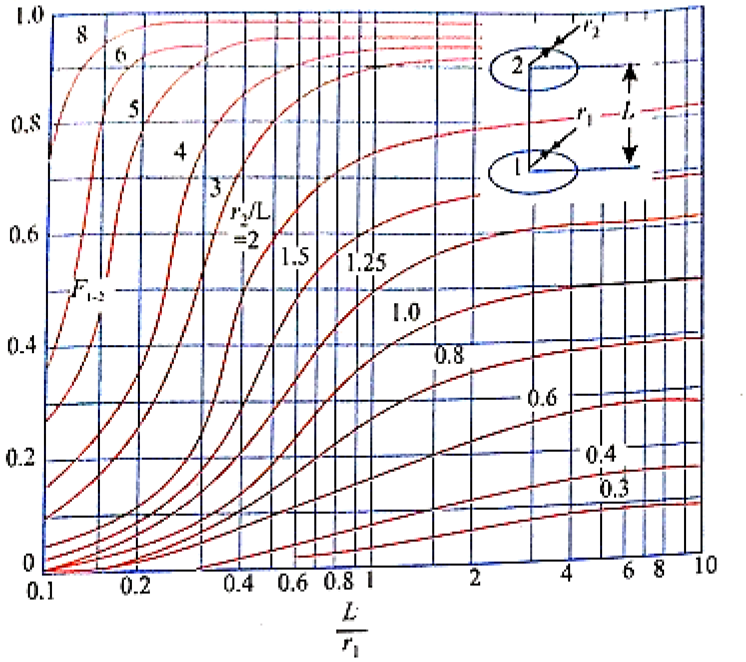


Fig. Shape factor for coaxial parallel discs.

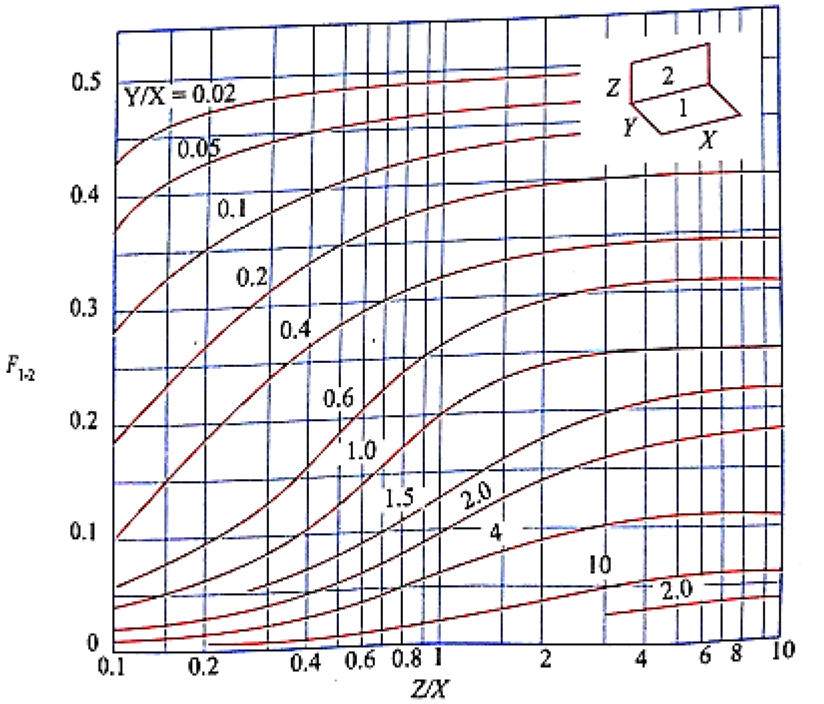


Fig. . Shape factor for perpendicular rectangles with a common edge.

LINK BELOW

https://www.youtube.com/watch?v=7_zv7g1c_U4

SHAPE FACTOR ALGEBRA

- In order to compute the shape factor for certain geometric arrangements for which shape factors or equations are not available , the concept of shape factor as fraction of intercepted energy and reciprocity theorem can be used .
- The shape factors for these geometries can be derived in terms of known shape factors of other geometries.
- The interrelation between various factors is called shape factor algebra.
- For the calculation of shape factors for specific geometries and for the analysis of radiant heat exchange between surfaces , the following facts and properties will be useful;

1. The shape factor is purely a function of geometric parameters only.
2. When two bodies are exchanging radiant energy with each other , the shape factor relation is given by

$$A_1 F_{1-2} = A_2 F_{2-1}$$

In general, $A_i F_{1-j} = A_j F_{j-i}$ (Reciprocity theorem)

This relation is useful when one of the shape factors is unity.

3. When all the radiation emanating from a convex surface 1 is intercepted by the enclosing surface 2 , the shape factor of convex surface with respect to the enclosure F_{1-2} is unity.Then with use of reciprocity theorem , the shape factor F_{2-1} is only the ratio of areas.

   i.e. when surface A_1 is entirely convex, say a sphere, completely enclosed by A_2 , then according to reciprocity theorem , we have

   $A_1 F_{1-2} = A_2 F_{2-1}$ and $A_1 = A_2 F_{2-1}$ (since F_{1-2}=1, as surface 1 completely sees surface 2)

Or, $F_{2-1} = \dfrac{A_1}{A_2}$ and $F_{2-1} + F_{2-2} = 1$

In this case , the black body radiation exchange is

$$Q_{12} = A_1 \sigma (T_1^4 - T_2^4)$$

4. A concave surface has a shape factor with itself because the radiant energy coming out from one part of the surface is intercepted by the another part of the same surface. The shape factor of a surface with respect to itself is F_{1-1}.
5. For a flat or convex surface , the shape factor with respect to itself is zero(i.e. F_{1-1}=0).This is due to the fact that for any part of flat or convex surface , one can not see any other part of the same surface.
6. If two surfaces A_1 and A_2 are parallel and large, radiation occurs across the gap between them so that A_1 and A_2 and all radiation emitted by one falls on the other, then

$$F_{1-2} = F_{2-1} = 1$$

7. If one of the two surfaces (say A_i) is divided into sub areas A_{i1}. A_{i2}..........A_{in}, then

$$A_i F_{i-j} = \sum A_{in} F_{in-j}$$

Referring to figure –a radiating surface A_1 has been split up into areas A_3 and A_4. We have

$$A_1 F_{1-2} = A_3 F_{3-2} + A_4 F_{4-2}$$

Hence, $$F_{1-2} \neq F_{3-2} + F_{4-2}$$

Thus if the radiant surface is subdivided , the shape factor for that surface with respect to the receiving surface is not equal to the sum of the individual shape factors.

Referring to figure-b , receiving surface A_2 has been divided into subareas A_3 and A_4, we have

$$A_1 F_{1-2} = A_1 F_{1-3} + A_1 F_{1-4}$$

Or, $$F_{1-2} = F_{1-3} + F_{1-4}$$

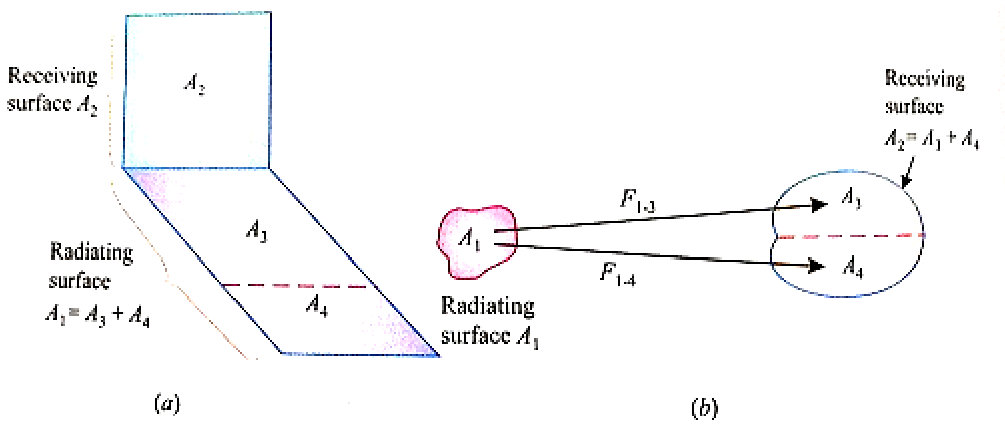


Fig. Relation between shape factors.

SHAPE FACTOR ALGEBRA –LINK BELOW

https://www.youtube.com/watch?v=YSZjEROVORE

HEAT EXCHANGE BETWEEN NON-BLACK BODIES

- The black body concept is an idealization which serves as a standard for the performance of a real body. In engineering applications, most surfaces do not behave like black bodies which absorb the entire incident radiation.
- The real surfaces (non-black) do not absorb the whole of the incident radiation: a part is reflected back to the radiating surface and this back and forth reflections between the surfaces may go on several times.
- As the emissivities and absorptivities are not uniform in all directions and for all wavelengths, it is required to simplify the problem to some extent by considering the bodies to be gray for which the emissivities and absorptivities are constant over the whole wavelength spectrum..

INFINITE PARALLEL PLANES

- The following assumptions are made for the analysis of radiant heat exchange between two non-black parallel surfaces;

  1. The configuration factor of either surface is unity.
  2. There is non-absorbing medium (such as air) in between the surfaces.
  3. The emissive and reflective properties are constant over all the surfaces.

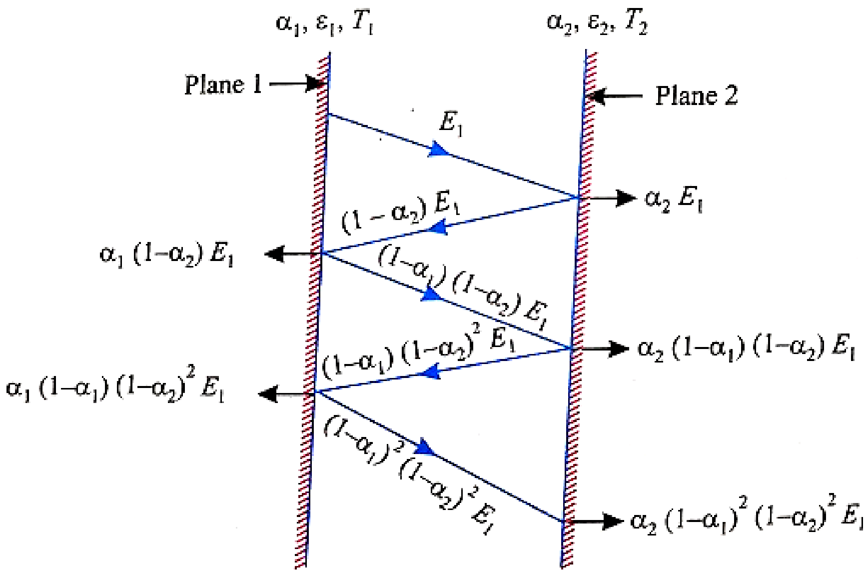


Fig. Heat exchange (radiant) between two non-black parallel surfaces.

- As shown in above figure , the surface (plane) 1 emits radiant energy E_1 which strikes the surface (plane) 2. Here a part $\alpha_2 E_1$ is absorbed (by surface 2) and $(1-\alpha_2)E_1$ (remainder) is reflected back to surface 1.
- Here a part $\alpha_1(1-\alpha_2)E_1$ is absorbed and remainder $(1-\alpha_1)(1-\alpha_2)E_1$ is reflected and so on.
- The amount of energy that has left surface 1 per unit time (Q_1)is given by:

$$Q_1 = E_1 - \left[\alpha_1(1-\alpha_2)E_1 + \alpha_1(1-\alpha_1)(1-\alpha_2)^2 E_1 + \alpha_1(1-\alpha_1)^2(1-\alpha_2)^3 E_1 + ...\right]$$

$$= E_1 - \alpha_1(1-\alpha_2)E_1\left[1 + (1-\alpha_1)(1-\alpha_2) + (1-\alpha_1)^2(1-\alpha_2)^2 + ...\right]$$

$$= E_1 - \alpha_1(1-\alpha_2)E_1[1 + Z + Z^2 + ...]$$

Where $Z = (1-\alpha_1)(1-\alpha_2)$

As Z<1 , the series $1+Z+Z^2+.....$, when extended to infinity gives 1/1-Z.

Hence, $Q_1 = E_1 - \dfrac{\alpha_1(1-\alpha_2)E_1}{1-Z} = E_1\left[1 - \dfrac{\alpha_1(1-\alpha_2)}{1-(1-\alpha_1)(1-\alpha_2)}\right]$

As per Kirchhoff's law , emissivity and absorptivity of a surface are equal and so $\alpha_1=\epsilon_1$ and $\alpha_2=\epsilon_2$.

Hence, $$Q_1 = E_1\left[1 - \frac{\varepsilon_1(1-\varepsilon_2)}{1-(1-\varepsilon_1)(1-\varepsilon_2)}\right]$$

$$= E_1\left[\frac{1-(1-\varepsilon_1)(1-\varepsilon_2) - \varepsilon_1(1-\varepsilon_2)}{1-(1-\varepsilon_1)(1-\varepsilon_2)}\right]$$

$$= E_1\left[\frac{1-(1-\varepsilon_2-\varepsilon_1+\varepsilon_1\varepsilon_2) - (\varepsilon_1-\varepsilon_1\varepsilon_2)}{1-(1-\varepsilon_2-\varepsilon_1-\varepsilon_1\varepsilon_2)}\right]$$

$$= E_1\left[\frac{1-1+\varepsilon_2+\varepsilon_1-\varepsilon_1\varepsilon_2-\varepsilon_1+\varepsilon_1\varepsilon_2}{1-1+\varepsilon_2+\varepsilon_1-\varepsilon_1\varepsilon_2}\right]$$

Or, $$Q_1 = \frac{E_1\varepsilon_2}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}$$

- Similarly the surface 2 emits radiant energy E_2 . A part $\alpha_1 E_2$ is absorbed by the surface 1 and the remainder $(1-\alpha_1)E_2$ is reflected back to surface 2. A part $\alpha_2(1-\alpha_1)E_2$ is absorbed and the rest $(1-\alpha_1)(1-\alpha_2)E_2$ is reflected and so on.

- Proceeding as done earlier , we can determine that the amount of radiant energy which leaves surface 2 per unit time is given by

$$Q_2 = E_2\left[\frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}\right]$$

- The net flow of heat from surface 1 to surface 2 per unit time is given by

$$Q_{12} = Q_1 - Q_2$$

$$= \frac{E_1\varepsilon_2}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2} - \frac{E_2\varepsilon_1}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}$$

$$= \frac{E_1\varepsilon_2 - E_2\varepsilon_1}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}$$

For non-black surfaces,

$E_1 = \varepsilon_1\sigma T_1^4$ and $E_2 = \varepsilon_2\sigma T_2^4$ ....By Stefan-Boltzmann law

- Hence, $$Q_{12} = \frac{\varepsilon_1\sigma T_1^4\varepsilon_2 - \varepsilon_2\sigma T_2^4\varepsilon_1}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}$$

Or, $$Q_{12} = \frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}\sigma(T_1^4 - T_2^4)$$

Or, $$Q_{12} = f_{1-2}\sigma(T_1^4 - T_2^4)$$

Where, $$f_{1-2} = \frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2-\varepsilon_1\varepsilon_2}$$

Or, $$f_{1-2} = \frac{1}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1}$$

And is called interchange factor for the radiation from surface 1 to surface 2.

INFINITE LONG CONCENTRIC CYLINDERS

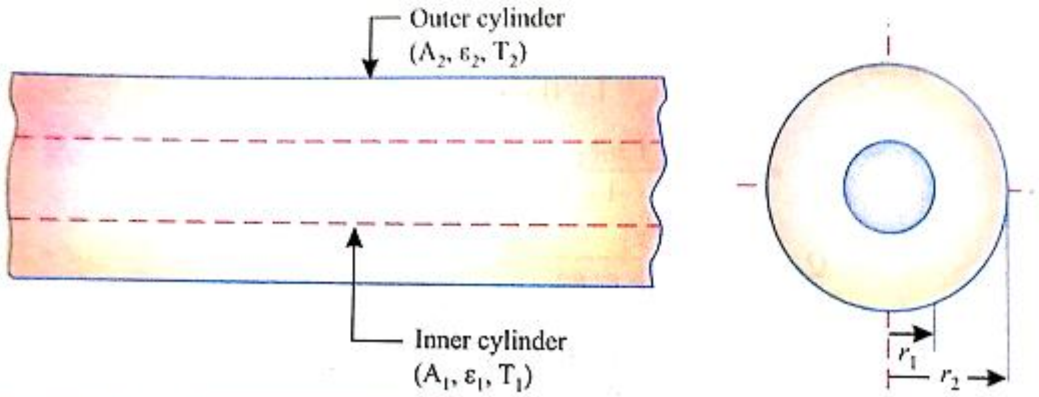


Fig. Heat exchange (radiant) between two infinite long concentric cylinders.

Consider two concentric cylinders as shown n figure above of area A_1 and A_2 , emissivities ϵ_1 and ϵ_2 and their surfaces maintained at temperatures T_1 and T_2 respectively.

Now, $$A_1F_{1-2} = A_2F_{2-1}$$

Or, $$F_{1-2} = 1$$

(Since all the radiations emitted by the inner cylinder are intercepted by the outer cylinder)

Hence $$F_{2-1} = \frac{A_1}{A_2}$$

Consider the energy emitted per unit area by the inner cylinder :

Inner cylinder emits energy=E_1

Outer cylinder absorbs energy =$\alpha_2 E_1 = \epsilon_2 E_1$ (since $\alpha_2=\epsilon_2$)

Outer cylinder reflects energy $= E_1 - \varepsilon_2 E_1 = E_1(1-\varepsilon_2)$

Inner cylinder absorbs energy $= E_1(1-\varepsilon_2)F_{2-1}\alpha_1 = E_1(1-\varepsilon_2)\dfrac{A_1}{A_2}\varepsilon_1$

Inner cylinder reflects energy $= E_1(1-\varepsilon_2) - E_1(1-\varepsilon_2)\varepsilon_1\dfrac{A_1}{A_2}$

$$= E_1(1-\varepsilon_2)\left[1-\varepsilon_1\frac{A_1}{A_2}\right]$$

It can be shown that the energy absorbed by the inner cylinder on the second reflection would be

$$= E_1(1-\varepsilon_2)^2\varepsilon_1\frac{A_1}{A_2}\left[1-\frac{A_1}{A_2}\varepsilon_1\right]$$

This absorption and reflection continue indefinitely , so we can find the net energy lost by the inner cylinder considering infinite times absorptions and reflections.

Hence, heat lost by the inner cylinder per unit area is given by

$$Q_1 = E_1 - E_1(1-\varepsilon_2)\varepsilon_1\frac{A_1}{A_2} - E_1(1-\varepsilon_2)^2\varepsilon_1\frac{A_1}{A_2}\left[1-\frac{A_1}{A_2}\varepsilon_1\right] + \ldots\ldots\ldots$$

$$= E_1\left[1-\frac{A_1}{A_2}\varepsilon_1(1-\varepsilon_2) - (1-\varepsilon_2)^2\varepsilon_1\frac{A_1}{A_2}\left\{1-\frac{A_1}{A_2}\varepsilon_1\right\} + \ldots\ldots\ldots\ldots\right]$$

$$= E_1\left[1-\frac{A_1}{A_2}\varepsilon_1(1-\varepsilon_2)\left\{1+(1-\varepsilon_2)\left(1-\frac{A_1}{A_2}\varepsilon_1\right)\right\}+................\right]$$

$$= E_1\left[1-\frac{A_1}{A_2}\varepsilon_1(1-\varepsilon_2)\left\{1-(1-\varepsilon_2)\left(1-\frac{A_1}{A_2}\varepsilon_1\right)\right\}^{-1}+................\right]$$

$$= E_1\left[1-\frac{\frac{A_1}{A_2}\varepsilon_2(1-\varepsilon_2)}{\left\{1-(1-\varepsilon_2)\left(1-\frac{A_1}{A_2}\varepsilon_1\right)\right\}}\right]$$

Or, $$Q_1=\frac{E_1\varepsilon_2}{\frac{A_1}{A_2}\varepsilon_1+\varepsilon_2-\frac{A_1}{A_2}\varepsilon_1\varepsilon_2}................(1)$$

Similarly, net energy lost by the outer cylinder per unit area would be,

$$Q_2=\frac{\varepsilon_1 E_2.\frac{A_1}{A_2}}{\frac{A_1}{A_2}\varepsilon_1+\varepsilon_2-\frac{A_1}{A_2}\varepsilon_1\varepsilon_2}................(2)$$

The net radiation heat transfer between the inner and outer concentric cylinders is given by

$$Q_1=Q_1-Q_2$$

$$= A_1\left[\frac{E_1\varepsilon_2}{\frac{A_1}{A_2}\varepsilon_1+\varepsilon_2-\frac{A_1}{A_2}\varepsilon_1\varepsilon_2}\right]-A_2\left[\frac{\varepsilon_1 E_2.\frac{A_1}{A_2}}{\frac{A_1}{A_2}\varepsilon_1+\varepsilon_2-\frac{A_1}{A_2}\varepsilon_1\varepsilon_2}\right]$$

$$=\frac{A_1E_1\varepsilon_2-A_1E_2\varepsilon_1}{\frac{A_1}{A_2}\varepsilon_1+\varepsilon_2-\frac{A_1}{A_2}\varepsilon_1\varepsilon_2}$$

Also, for non black bodies

$$E_1=\varepsilon_1\sigma T_1^4 and E_2=\varepsilon_2\sigma T_2^4$$

$$Q_{12}=\frac{A_1\varepsilon_1\varepsilon_2\sigma T_1^4-A_1\varepsilon_1\varepsilon_2\sigma T_2^4}{\frac{A_1}{A_2}\varepsilon_1+\varepsilon_2-\frac{A_1}{A_2}\varepsilon_1\varepsilon_2}$$

$$= \frac{\varepsilon_1 \varepsilon_2 A_1 \sigma (T_1^4 - T_2^4)}{\left[\frac{A_1}{A_2} \varepsilon_1 \varepsilon_2 \left(\frac{1}{\varepsilon_2} - 1\right)\right] + \varepsilon_2}$$

Or, $$Q_{12} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)} = f_{1-2} A_1 \sigma (T_1^4 - T_2^4) \text{......................}(3)$$

Where, $$f_{1-2} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

Is the interchange factor or equivalent emissivity for radiant heat exchange between infinite long concentric cylinders.

SMALL GRAY BODIES

Consider two small gray bodies (the gray bodies are said to be small if their size is very small as compared to the distance between them) 1 and 2 having emissivities ϵ_1, ϵ_2 or absorptivities α_1 , α_2.

The remitted radiant energy emitted by body 1 is partly absorbed by body 2. The portion of radiation unabsorbed and thus reflected on the first incidence is considered to be lost in space (due to surfaces being small) i.e., nothing returns back to surface 1.

Similar is the case with surface 2.

The energy emitted by body 1 $= A_1 \varepsilon_1 \sigma T_1^4$

The energy incident on body 2 $= F_{1-2} A_1 \varepsilon_1 \sigma T_1^4$

The energy absorbed by body 2 $= \alpha_2 F_{1-2} A_1 \varepsilon_1 \sigma T_1^4$

The energy transfer from body 1 to body 2,

$$Q_1 = \varepsilon_1 \varepsilon_2 A_1 F_{1-2} \sigma T_1^4$$ Since, $\alpha_2 = \epsilon_2$

Similarly the energy transfer from body 2 to body 1,

$$Q_2 = \varepsilon_1 \varepsilon_2 A_2 F_{2-1} \sigma T_2^4$$

The net radiant heat exchange between the two bodies is

$$Q_{12} = \varepsilon_1 \varepsilon_2 A_1 F_{1-2} \sigma T_1^4 - \varepsilon_1 \varepsilon_2 A_2 F_{2-1} \sigma T_2^4$$

But $$A_1 F_{1-2} = A_2 F_{2-1}$$

Hence, $$Q_{12} = \varepsilon_1 \varepsilon_2 A_1 F_{1-2}(T_1^4 - T_2^4)$$

$$= f_{1-2} A_1 F_{1-2}(T_1^4 - T_2^4)$$

Where , $f_{1-2}=\epsilon_1\epsilon_2$ represents the equivalent emissivity or interchange factor for radiant heat exchange between two small gray bodies.

SMALL BODY IN A LARGE ENCLOSURE

Consider a small body placed in large enclosure . The large gray enclosure acts like a black body , absorbing practically all the radiation incident upon it and reflecting negligibly small energy back to the small gray body .

In this case $F_{1-2}=1$ since all the radiations emitted by the small body would be intercepted by the outer large enclosure . Thus,

Energy emitted by small body 1 and absorbed by the outer large enclosure 2 $= A_1 \varepsilon_1 \sigma T_1^4$

Energy emitted by the enclosure 2 $= A_2 \varepsilon_2 \sigma T_2^4$

Energy incident upon the small body 1 $= F_{2-1} A_2 \varepsilon_2 \sigma T_2^4$

Energy absorbed by the small body 1 $= \alpha_1 F_{2-1} A_2 \varepsilon_2 \sigma T_2^4 = \varepsilon_1 \varepsilon_2 A_2 F_{2-1} \sigma T_2^4$

[Since, $\alpha_1=\epsilon_1$]

Hence, the net radiant heat exchange between the small body 1 and the outer large enclosure 2,

$$Q_{12} = \varepsilon_1 A_1 \sigma T_1^4 - \varepsilon_1 \varepsilon_2 A_2 F_{2-1} \sigma T_2^4$$

If $T_1=T_2$ and $Q_{12}=0$, we have

$$A_1 = A_2 \varepsilon_2 F_{2-1} \quad \text{and}$$

$$Q_{12} = \varepsilon_1 A_1 \sigma (T_1^4 - T_2^4)$$

$$= f_{1-2} A_1 \sigma (T_1^4 - T_2^4)$$

Where, f_{1-2} represents the equivalent emissivity or interchange factor for radiation heat exchange between a small body and a large enclosure.

Net radiation heat exchange between two gray surfaces , considering both the interchanging factor f_{1-2} and geometric factor F_{1-2}, is given by,

$$Q_{net} = f_{1-2} F_{1-2} \sigma A_1 (T_1^4 - T_2^4)$$

ELECTRICAL NETWORK ANALOGY FOR THERMAL RADIATION SYSTEMS

- An electrical network analogy is an alternative approach for analyzing radiation heat exchange between gray or black surface. In this approach the two terms commonly used are irradiation and radiosity.
- **Irradiation(G)-** It is defined as the total radiation incident upon a surface per unit time per unit area. It is expressed in W/m^2.
- **Radiosity-** This term is used to indicate the total radiation leaving a surface per unit time per unit area. It is also expressed in W/m^2. The radiosity comprises the original emittance from the surface plus the reflected portion of any radiation incident upon it.

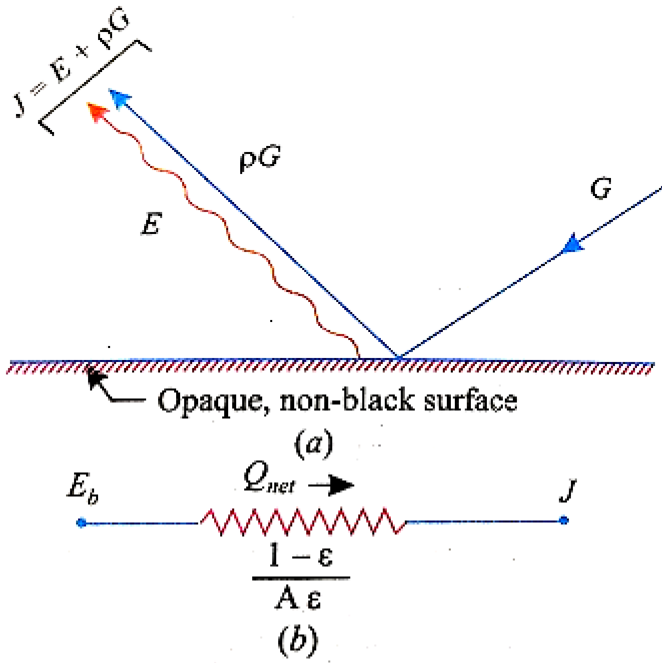


Fig. Irradiation and radiosity.

i.e., $J=E+\rho G$

or, $J=\epsilon E_b+Pg$……………………..(1)

Also, $\alpha+\rho+\tau=1$

Or, $\alpha+\rho=1$ ($\tau=0$, the surface being opaque)

Or, $\rho=1-\alpha$

Hence, $J=\epsilon E_b+(1-\alpha)G$

But, $\alpha=\epsilon$

Hence, $J=\epsilon E_b+(1-\epsilon)G$…………………………………(2)

Or, $J-\epsilon E_b=(1-\epsilon)G$

Or, $$G = \frac{J - \varepsilon E_b}{1-\varepsilon} \text{.......................(3)}$$

The net energy leaving a surface is the difference between its radiosity and irradiation. Thus,

$$\frac{Q_{net}}{A} = J - G\text{.......................(4)}$$

Or, $$\frac{Q_{net}}{A} = J - \frac{J - \varepsilon E_b}{1-\varepsilon} = \frac{J(1-\varepsilon) - (J - \varepsilon E_b)}{1-\varepsilon}$$

$$= \frac{J - J\varepsilon - J + \varepsilon E_b}{1-\varepsilon} = \frac{\varepsilon (E_b - J)}{1-\varepsilon}$$

Or, $$Q_{net} = \frac{A\varepsilon (E_b - J)}{1-\varepsilon} = \frac{E_b - J}{(1-\varepsilon)/A\varepsilon} \text{...............(5)}$$

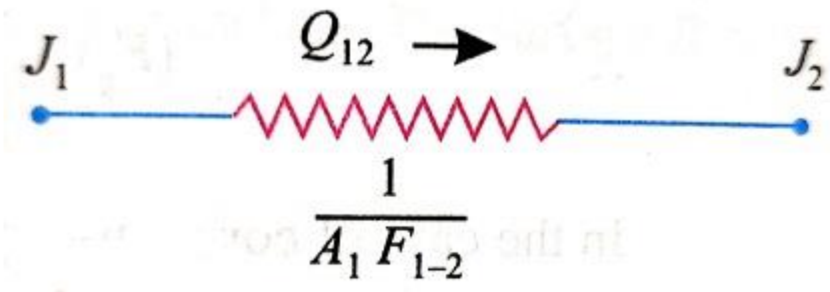
- The representation of this equation in the form of electric network is shown in figure-b. The quantity 1-ϵ/Aϵ is known as surface resistance, as it is related to surface properties of the radiating body.
- Now consider the exchange of radiant energy between the two surfaces (non-black) 1 and 2. Of the total radiation which leaves surface 1 , the amount that reaches the surface 2 is $J_1A_1F_{1-2}$.
- Similarly , the heat radiated by surface 2and received by surface 1 is $J_2A_2F_{2-1}$.
- The net interchange of heat between the surfaces is given by

$$Q_{12} = J_1 A_1 F_{1-2} - J_2 A_2 F_{2-1}\text{................(6)}$$

But, $$A_1 F_{1-2} = A_2 F_{2-1}$$

Hence, $$Q_{12} = A_1 F_{1-2}(J_1 - J_2)$$

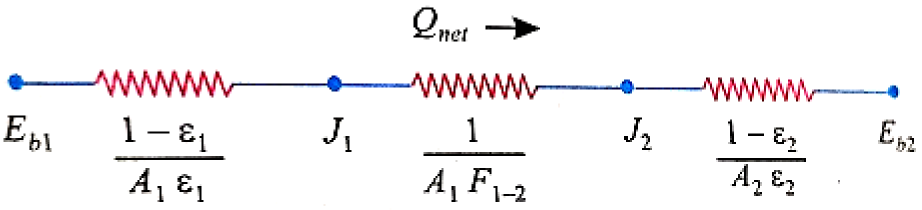
Or, $$Q_{12} = \frac{J_1 - J_2}{1/A_1 F_{1-2}}\text{..................(7)}$$



This equation -7 can be represented in the form of electric network , The quantity $1/A_1F_{1-2}$ is called the space resistance because it is due to the distance and geometry of the radiating bodies.

If the surface resistances of the two bodies and space resistance between them is considered then the net heat flow can be represented by an electric circuit.

The net heat exchange between the two gray surfaces is given by



$$(Q_{12})_{net}=\frac{E_{b_1}-E_{b_2}}{\frac{1-\varepsilon_1}{A_1\varepsilon_1}+\frac{1}{A_1F_{1-2}}+\frac{1-\varepsilon_2}{A_2\varepsilon_2}}=\frac{A_1\sigma(T_1^4-T_2^4)}{\frac{1-\varepsilon_1}{\varepsilon_1}+\frac{1}{F_{1-2}}+\frac{1-\varepsilon_2}{\varepsilon_2}.\frac{A_1}{A_2}}$$

Or, $$(Q_{12})_{net}=(F_g)_{1-2}A_1\sigma(T_1^4-T_2^4)\ldots\ldots\ldots\ldots\ldots(8)$$

Where, $$(F_g)_{1-2}=\frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1}+\frac{1}{F_{1-2}}+\frac{1-\varepsilon_2}{\varepsilon_2}.\frac{A_1}{A_2}}$$

And is known as gray body factor.

When exchange of heat takes place between two black surfaces, the surface resistances becomes zero as $\epsilon_1=\epsilon_2=1$: $(F_g)_{1-2}$ chages to F_{1-2} equation-8 reduces to

$$(Q_{12})_{net}=F_{1-2}A_1\sigma(T_1^4-T_2^4)\ldots\ldots\ldots\ldots\ldots\ldots\ldots \textit{forblacksurfaces}$$

Let us consuder the following cases:

1. When the radiating bodies are infinite parallel;

   In this case $A_1=A_2$ and $F_{1-2}=1$

$$(F_g)_{1-2}=\frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1}+1+\frac{1-\varepsilon_2}{\varepsilon_2}}=\frac{1}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1}\ldots\ldots\ldots\ldots(9)$$

2. When the radiating bodies are concentric cylinders or spheres:

   Here $F_{1-2}=1$

$$(F_g)_{1-2} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + 1 + \frac{1-\varepsilon_2}{\varepsilon_2} \cdot \frac{A_1}{A_2}} \ldots\ldots\ldots\ldots\ldots(10)$$

In the case of concentric cylinders,

$$\frac{A_1}{A_2} = \frac{\pi d_1 l}{\pi d_2 l} = \frac{d_1}{d_2} = \frac{r_1}{r_2} \ldots\ldots\ldots\ldots\ldots\ldots(11)$$

In the case of concentric spheres,

$$\frac{A}{A_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

3. When a small body lies inside a large enclosure:

Here, $F_{1\text{-}2}=1$. $A_1 << A_2$, so that $\frac{A_1}{A_2} \to 0$

Hence, $$(F_g)_{1-2} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + 1} \ldots\ldots\ldots\ldots\ldots\ldots(13)$$

GRAY BODY FACTOR

https://www.youtube.com/watch?v=qdCE5i-UG2k

RADIATION SHIELDS

- In some cases it is required to reduce the overall heat transfer between two radiating surfaces. This is done by either using materials which are highly reflective or by using radiation shields between the heat exchanging surfaces.
- The radiation shields reduce the radiation heat transfer by effectively increasing the surface resistances without actually removing any heat from the overall system.
- Thin sheets of plastic coated with highly reflecting metallic films on both sides serve as very effective radiation shields.
- These are used for the insulation of cryogenic storage tanks.
- A familiar application of radiation shields is in the measurement of the temperature of a fluid by a thermometer or a thermocouple which is shielded to reduce the effects of radiation.
- Let us consider two parallel plates, 1 and 2 , each of area A at temperatures T_1 and T_2 respectively with a radiation shield placed between them as shown in the figure .

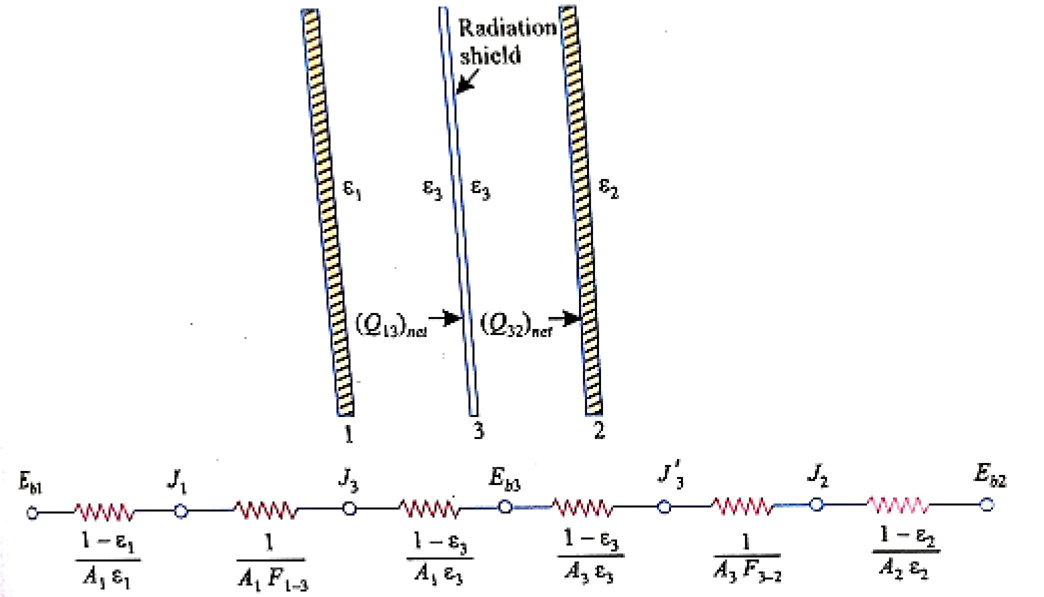


Fig. Radiation network for two parallel infinite planes separated by one shield.

- With no radiation shields, the net heat exchange between the parallel plates is given by:

$$(Q_{12})_{net} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1} \text{..............(1)}$$

- If the emissivity of the radiation shield is ϵ_3, we can use this equation to find heat exchange between surfaceas 1,3 and 3,2.

$$(Q_{13})_{net} = \frac{A\sigma(T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_3}-1} \text{.................(2)}$$

$$(Q_{32})_{net} = \frac{A\sigma(T_3^4 - T_2^4)}{\frac{1}{\varepsilon_3}+\frac{1}{\varepsilon_2}-1} \text{..................(3)}$$

- Since the radiation shield does not deliver or remove heat from the system, therefore,

$$(Q_{13})_{net} = (Q_{32})_{net}$$

$$\frac{A\sigma(T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_3}-1} = \frac{A\sigma(T_3^4 - T_2^4)}{\frac{1}{\varepsilon_3}+\frac{1}{\varepsilon_2}-1} \text{.................................(4)}$$

Simplifying of above equation gives

$$T_3^4 = \frac{T_1^4\left(\frac{1}{\varepsilon_3}+\frac{1}{\varepsilon_2}-1\right)+T_2^4\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_3}-1\right)}{\left(\frac{1}{\varepsilon_3}+\frac{1}{\varepsilon_2}-1\right)+\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_3}-1\right)}\text{..............}(5)$$

Substituting the value of T_3 in the left hand side of equation-4 , we get

$$(Q_{12})_{net} = \frac{A\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\varepsilon_3}+\frac{1}{\varepsilon_2}-1\right)+\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_3}-1\right)}\text{....................}(6)$$

Hence, $$\frac{[(Q_{12})_{net}]_{wuthshield}}{[(Q_{12})_{net}]_{wuthoutshield}} = \frac{\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_2}-1}{\left(\frac{1}{\varepsilon_1}+\frac{1}{\varepsilon_3}-1\right)+\left(\frac{1}{\varepsilon_3}+\frac{1}{\varepsilon_2}-1\right)}\text{....................}(7)$$

If $\epsilon_1=\epsilon_2=\epsilon_3$, then right hand side of above equation,

$$\frac{1}{2} or (Q_{13})_{net} = (Q_{32})_{net} = \frac{1}{2}(Q_{12})_{net}$$

Thus when one shield is inserted between two parallel surfaces, the direct radiation heat transfer between them is halved. The corresponding value of T_3 of the shield attains the value

$$T_3^4 = \frac{1}{2}(T_1^4 + T_2^4)\text{.................}(8)$$

In the general case where there are n shields, all the surface resistances would be the same , since the emissivites are equal. There will be two surface resistances for each shield and one for each heat transfer surface. There will also be (n+1) space resistances but the configuration factor is unity for each infinite parallel plane.

Total resistance $$(R)_{n-shields} = \left[(2n+2)\left(\frac{1-\varepsilon}{\varepsilon}\right)+(n+1)(1)\right]/A$$

$$= \left[(n+1)\left(\frac{2}{\varepsilon}-1\right)\right]/A\text{..................}(9)$$

The radiant heat transfer rate between two infinitely large parallel plates separated by n –shields is therefore,

$$(Q)_{n-shields} = \frac{1}{(n+1)\left(\frac{2}{\varepsilon}-1\right)}A\sigma(T_1^4 - T_2^4)\text{...............}(10)$$

Where n=0. i.e no shields , the resistance is given by

$$(R)_{withoutshield} = \left[\frac{2}{\varepsilon} - 1\right] / A \text{.................}(11)$$

And thus,
$$(Q)_{withutshield} = \frac{A\sigma(T_1^4 - T_2^4)}{\left(\frac{2}{\varepsilon} - 1\right)} \text{.............}(12)$$

So we get,
$$\frac{(Q)_{n-shields}}{(Q)_{withoutshield}} = \frac{(R)_{withoutshield}}{(R)_{n-shields}} = \frac{1}{n+1} \text{............}(13)$$

Arranging equation-6, in a different form, we get

$$(Q_{12})_{net} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + 2\left(\frac{1}{\varepsilon_3}\right) - 2}$$

This equation can be generalized for a system of two parallel plates separted by n-shields of emissivity ϵ_{s1}, ϵ_{s2}...,,,,,ϵ_{sn} as

$$(Q_{12})_{net} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + 2\sum_{i=1}^{n}\frac{1}{\varepsilon_{si}} - (n+1)} \text{......................}(14)$$

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